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***Public Debt Indexation and Denomination,
The Case of Brazil: A Comment***

Rubens Penha Cysne

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Public Debt Indexation and Denomination, The Case of Brazil: A Comment*

Rubens Penha Cysne[†]

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Abstract

In this work I analyze the model proposed by Goldfajn (2000) to study the choice of the denomination of the public debt. The main purpose of the analysis is pointing out possible reasons why new empirical evidence provided by Bevilaqua, Garcia and Nechio (2004), regarding a more recent time period, finds a lower empirical support to the model. I also provide a measure of the overestimation of the welfare gains of hedging the debt led by the simplified time frame of the model. Assuming a time-preference parameter of 0.9, for instance, welfare gains associated with a hedge to the debt that reduces to a half a once-for-all 20%-of-GDP shock to government spending run around 1.43% of GDP under the no-tax-smoothing structure of the model. Under a Ramsey allocation, though, welfare gains amount to just around 0.05% of GDP.

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[†]Professor of Economics at the Graduate School of Economics of the Getulio Vargas Foundation (EPGE/FGV) and a Visiting Scholar at the Department of Economics of the University of Chicago.

1 Introduction

Among the different approaches used by economic theorists regarding the management of public debt there are those which concentrate on time-consistency (e.g., Calvo (1978) and Lucas and Stokey (1983)), those which focus on exogenous (government-led) tax smoothing (Barro (1979) being the original and main reference in this group) and those which focus on endogenous (covariance-led) tax smoothing (e.g., Bohn(1988, 1990a, 1990b), Goldfajn (1996, 2000) and Miller (1997)).

On the one side (Barro's work left aside), the time-consistency approach argues that indexing the debt to domestic price indexes has the advantage of avoiding the temptation of future governments to reduce the real value of the liabilities of the public sector by increasing the cost of living.

On the other side, the endogenous (covariance-led) tax-smoothing approach calls for issuing securities the returns of which are negatively correlated with the tax needs of the government. This side makes a point for nominal debt, if government expenditures are positively correlated with the rate of inflation; and/or a point for foreign-currency indexed debt, if the real exchange rate (defined in terms of the foreign-exchange price of the domestic currency) happens to be positively correlated with government spending.

The original argument of the hedging approach can be found in Bohn (1988), who argues:

"Intuitively, if government does not hedge, it has to vary taxes depending on the state of nature. But because of increasing marginal cost, welfare gains in good states (that allow tax cuts) are more than offset by welfare losses in bad states (that require tax increases). Nominal debt allows the government to hedge against bad states of the world at (close to) fair odds and to change taxes very little."

Goldfajn (1996, 2000) presents an original and easily readable one-period model¹ which delivers the type of result detailed in the paragraph above. Goldfajn tests his model using monthly data for Brazil covering the period from 1980 to 1997. The main empirical issue is investigating if the relative

¹Although the author refers to two periods (I do the same below, for didactic reasons), the model is actually a one-period model, since there is no discounting and since government's budget constraint and consumer's welfare are considered in only one period.

shares of each one of the components of the Brazilian public debt have followed the pattern suggested by the model. More recently, the same model and the same empirical methodology suggested by Goldfajn has been used by Goldfajn and De Paula (2000) and Bevilaqua, Garcia and Nechio (2004) to analyze the Brazilian experience with indexed debt.

Goldfajn's analysis does not include questions related to the costs of the debt, but solely with the variability of taxes². A higher variance of tax rates reduces welfare. By these means, hedging against shocks to the budget can be welfare improving.

This work is divided as follows. Section 2 points out possible reasons for the low empirical support of the model found by Bevilaqua, Garcia and Nechio (2004). The analysis focus on the simplified one-period time frame of Goldfajn's model. First, I argue that long-run, rather than one-period-ahead measures of government spending should be considered. Second, I claim that a more encompassing model should explicitly recognize the discrepancy between the time period economic agents have to acquire information (one month in the empirical measurements) and the maturity of the assets issued by the government. Failure of the model to address these issues is suggested, non exclusively, as possible reasons for its low empirical support.

Section 3 is used to illustrate that a one period-model tends to over-estimate possible gains derived from the hedging argument. The possibility of dynamically optimizing the distribution of taxes reduces the welfare losses of taxation and, consequently, the possible welfare gains of hedging.

2 The Model And The Empirical Evidence

Goldfajn's model aims at providing a simple structure to capture the hedging function provided by the nominal debt when inflation is positively correlated with government spending. A similar argument for foreign-exchange denominated debt can also be made.

²As the author writes in page 51: "In reality, governments claim that they manage debt to 'minimize the borrowing cost'. However, if markets work efficiently and there is no free lunch, any gains from shifting to cheaper securities should imply higher risks to the government. Since higher risks to the government imply, ultimately, higher risks to the society (for example with a higher probability of raising taxes to close the budget), it is not clear that there are any gains from this strategy."

To simplify the formal explanation of the model, I detail here only the case in which there is commitment. The objective of the government is to minimize the expected value of the distortions from taxes (τ) and from inflation (π):

$$\text{Min } E \left[A \frac{\tau^2}{2} + \frac{\pi^2}{2} \right] \quad (1)$$

where $A > 0$ and E denotes the expectation operator. (1) assumes the distortions from both taxes and inflation to be quadratic in these variables. The best way to interpret equation (1), regarding the distortions from taxation, is in terms of deviations from a previous levels $\bar{\tau}$ (to simplify, assumed to be 0 in (1)), in which case temporary oscillations of the tax rate decrease welfare (see equation (1) in Barro (1997) which, implicitly, makes a point about the distinction between levels and variations).

As pointed out by Barro (1997), the meaning of the term τ^2 in (1) is that variations in taxes over time cause distortions that the government would like to avoid.

In the model, government spending, real exchange rate and inflation are stochastic. In the first period, the government chooses the composition of the debt that it sells to the public and that matures in period 2. The budget constraint in the second period, on which I shall concentrate here, reads:

$$\tau = G + (1 + r)B(1 - \theta(\pi - \pi^e) - \theta^*(q - q^e)) \quad (2)$$

In equation (2), r is the real interest rate, θ and θ^* are the proportions of nominal and foreign denominated debt, respectively, B is the level of total debt, G is government spending and $q = \pi + \alpha$, α the nominal rate of appreciation of the domestic currency (here, price of the domestic currency). The superindex e denotes the expected value³.

The (constrained) minimization problem given by (1) and (2) leads to the optimal shares of nominal (θ) and exchange-rate-denominated (θ^*) debt:

$$\theta = \frac{\sigma_{g\pi}\sigma_q^2 - \sigma_{gq}\sigma_{q\pi}}{(1 + r)B(\sigma_\pi^2\sigma_q^2 - \sigma_{q\pi}^2)} \quad (3)$$

³The budget constraint (2) does not contain, as it should, the inflation-tax term on the right side. This point is acknowledged by the author in footnote 8 of the paper. As noted by Calvo and Guidotti (1992), the inclusion of the inflation tax modifies the conclusions of the model even when there is full commitment, since in this case it is not straightforward that $\pi = 0$.

$$\theta^* = \frac{\sigma_{gq}\sigma_{\pi}^2 - \sigma_{q\pi}\sigma_{g\pi}}{(1+r)B(\sigma_{\pi}^2\sigma_q^2 - \sigma_{q\pi}^2)} \quad (4)$$

where σ stands for variance or covariance of the respective subscripts. Note that the optimal proportion of the debt in nominal terms increases with the covariance of inflation and government spending and decreases with the variance of inflation. Foreign-exchange denominated debt increases with the covariance of inflation and with the covariance of government spending and the real exchange rate (defined as mentioned before).

- The Empirical Evidence With Brazilian Data

At least two sets of empirical evidence are now available regarding the predictions of this model. Goldfajn (2000) uses Brazilian data from 1980 to 1997, whereas Bevilaqua, Garcia and Nechio (2004) cover the period 1994-2003.

Goldfajn (2000) offers his model as an explanation of the fact that, in the aftermath of the Real Plan, the share of public indexed debt in Brazil dropped from 70 to 30% of total debt, while both nominal and foreign denominated debt shares have increased. His empirical findings are summarized in the following way:

"the evidence from ordinary least-squares (OLS) regressions confirms that the variance of inflation, the size of the public debt, and the correlations of inflation with spending are important determinants of public debt indexation in Brazil.

However, neither credibility nor the hedging motive is able to explain the proportion issued of foreign denominated debt."

New evidence found by Bevilaqua, Garcia and Nechio (2004), concerning a more recent period, presents a lower empirical support for Goldfajn's model. In particular, Table 3 in Bevilaqua et al., where regressions have as the dependent variable the share of the nominal public debt, shows a negative sign for the variance of inflation (though statistically non-significant) and a negative sign (this one statistically significant) for the covariance between inflation and government spending. Both signs are contrary to those that the model would predict. These authors conclude:

"in general, even though some assumptions of the model have been confirmed, and some signs have met what the model would predict, the results do not perfectly corroborate the model; other points not captured by the model should be taken into consideration".

3 Main Points

By endowing the model with just two (or one, see footnote 1) periods, the author is compelled to assume that the budget constraint (2) necessarily closes without resort to the issuance of new debt, and that all debt has a one period maturity. For this reason, the administration of the debt is supposed by the model to take actions wholly based on one-period-ahead variances and covariances and to deal only with a one-period maturity debt.

3.1 Government Spending and Planning Horizon

In this subsection I argue that forecasts based on conditional covariances of one-period-ahead government spending and inflation (or exchange rate) miss important informations related to the dynamic administration of the debt⁴.

Consider the original model with an infinite number of periods. The government is supposed not to be borrowing constrained, and to minimize the expected present value of distortions using a discount rate α ($0 < \alpha < 1$, V standing for the discounted value of distortions and E_t for the conditional expectation at time t):

$$V(\tau_t, B_t) = E_t \sum_{j=t+1}^{\infty} g^j \left(A \frac{(\tau_j - \bar{\tau}_t)^2}{2} + \frac{\pi_j^2}{2} \right) \quad (5)$$

Assume that the economy departs from a steady in period t with:

$$\tau_t = G_t + (1 + r_t)B_t(1 - \theta_t(\pi_t - \pi_t^e) - \theta_t^*(q_t - q_t^e)) \quad (6)$$

and follows the equation of motion:

$$\tau_{t+1} - \tau_t = G_{t+1} + (1 + r_t)B_t(1 - \theta_t(\pi_t - \pi_t^e) - \theta_t^*(q_t - q_t^e)) - \tau_t - \Delta_{t+1} \quad (7)$$

with $\Delta_{t+1} = B_{t+1} - B_t$ denoting the issuance of new debt.

I consider a simple example that illustrates the point to be made. Suppose that some expenditures of the government, previously forecasted to be incurred two periods ahead, are now forecasted to be incurred in one period. Suppose that a proper discount rate is used to transform the respective

⁴By referring to the innovations to the "permanent government spending", as opposed to innovations of the one-period-ahead government spending, as Goldfajn (2000) does, Missale (1999, p. 143) makes a point similar to the one I make here.

values. By definition, this operation does not change the present value of government spending and, in a dynamic optimizing setting, should not lead to any change of taxation. The new one-period-ahead positive shock to the government spending is completely offset in the next period.

In this case, minimization of (5) under commitment leads (as an approximation) in all periods to $\pi_t = 0$ and $\tau_t - \bar{\tau} = 0$, by simply having the government make, once the shock is known, in period $t + 1$:

$$\Delta_{t+1} = G_{t+1} - G_t \quad (8)$$

A negatively symmetric operation in period $t + 2$ allows the debt to remain payable in the long run, no changes in taxation being necessary. Departing from τ_t , the government can, therefore, keep taxes unaltered in this case. The administration of the public debt, acting optimally, would have no incentive to change the composition of the public debt, in an attempt to reduce the variations of the tax rates, for the simple reason that the tax rates would not have to change.

However, any model focusing on one-period ahead only would predict an increase in taxes and, by these means, an (incorrect) incentive of the administration of the debt to devise a hedging action. Assuming a negative correlation between inflation and government spending, Goldfajn's model in this case would have predicted the share of nominal debt to have increased in period t , whereas a deeper analysis of the situation shows that such an action would not be optimal.

Of course, there is nothing special with this example. The main point is that the data will always reflect actions taken by the administration of the debt based on an expected long-run profile of government spending, not based on a one-period-ahead government spending, as assumed by the model.

This type of contrast between the predictions of the model and the actions taken by the administration of the debt can be a source of low empirical support by the data.

- The Same Point From Another Angle

As another way to see the same point, more closely following the type of calculations done by Goldfajn, suppose the government wants to minimize (1) in period $t - 1$ taking Δ_t as unknown (indeed the government only knows

how much debt it has to issue after the shocks are realized). Then the minimization problem would lead to the normal equations:

$$E_{t-1}([G_t + (1 + r_t)B_t(1 - \theta_t(\pi_t - \pi_t^e) - \theta_t^*(q_t - q_t^e)) - \Delta_t] [\pi_t - \pi_t^e]) = 0 \quad (9)$$

and

$$E_{t-1}([G_t + (1 + r_t)B_t(1 - \theta_t(\pi_t - \pi_t^e) - \theta_t^*(q_t - q_t^e)) - \Delta_t] [q_t - q_t^e]) = 0 \quad (10)$$

These equations lead to expressions of the same form of (3) and (4), but with $\sigma_{(g-\Delta)\pi}$ and $\sigma_{(g-\Delta)q}$ substituting for, respectively, $\sigma_{g\pi}$ and σ_{gq} . Since these covariances are determined by government policies, θ and θ^* can actually assume any value.

3.2 Debt Maturity and Planning Horizon

In Goldfajn's model, the time frame for authorities and economic agents to modify their information set is (in the empirical evaluations) one month. However, this time period is not necessarily same as the maturity of the debt. For instance, in July of 2004, the average maturity of the domestic federal debt was around 30 months. This generates two types of contrasts between Goldfajn's predictions and the real data.

First, the model evaluated with monthly data implies that, each month, authorities would be willing to modify the composition of the whole stock of debt, based on the possibility of providing a hedge to it. This would demand maturities of one month, which is not the case, or massive repurchases of the debt each month, a too costly alternative.

If the effective maturity of the debt is equal to n months, $n > 1$, and if one assumes it to have a uniformly staggered structure, only a fraction around $1/n$ of it would be subject to optimization each month.

Second, the planning horizon of the optimization problem detailed in the previous section should take the proper maturity of the debt instrument into consideration. Goldfajn's model assumes an optimization based on a one-month horizon. If the forecasted (or desired) maturity of the debt is equal to n periods, though, this is the horizon which should be considered.

This second problem can be posed in terms of the impulse-response function for a VAR. Using this framework is also useful here because the empirical evaluations of Goldfajn's model are carried out in terms of one-period-ahead conditional covariances calculated with the use of VARs.

Imagine that the variables in the problem under consideration here (x) relate to each other dynamically through a set of stochastic difference equations represented in MA (∞) form by⁵:

$$x_t = B(L)\varepsilon_t, \quad B(0) = I, \quad E(\varepsilon_t \varepsilon_t') = \Sigma \quad (11)$$

Suppose that the administration of the debt minimizes the unpredictable variation of the tax needs of the government, $\tau_t - E_{t-1} \tau_t$, and government spending and inflation covary positively. Because part of the debt is nominal, an increase in inflation reduces the real value of the government debt. Now suppose that there is a positive shock to government spending.

To simplify the exposition, consider a subsystem of (11) including only government spending g and inflation π ($x = (g, \pi)$), and imagine that the chosen Cholesky decomposition of Σ ($\Sigma = CC'$), which I call C , is such that g has a contemporaneous effect over π , but not vice-versa. Make:

$$C\eta_t = \varepsilon_t \quad (12)$$

Then $E(\eta_t \eta_t') = I$. Using (12) in (11), the 2×2 system can be expressed in terms of the orthogonalized errors as:

$$x_t = B(L)C\eta_t, \quad B(0) = I, \quad E(\eta_t \eta_t') = I \quad (13)$$

By writing:

$$\begin{bmatrix} g_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} c_{gg} & 0 \\ c_{g\pi} & c_{\pi\pi} \end{bmatrix} \begin{bmatrix} \eta_{gt} \\ \eta_{\pi t} \end{bmatrix} + C_1 \eta_{t-1} + \dots$$

one can easily see that an orthogonal shock η_{gt} to g leads to a contemporaneous response of inflation given by $E_t \pi_t - E_{t-1} \pi_t = c_{g\pi} \eta_{gt}$. Now observe that a positive covariance between g and π in the original covariance matrix ($\Sigma_{2,1} > 0$), as assumed, implies $c_{g\pi} > 0$. Indeed, by the Cholesky decomposition, $c_{g\pi} = \Sigma_{2,1}/c_{gg}$, where $c_{gg} = \sqrt{\Sigma_{1,1}} > 0$. Hence, when government spending increases, so does the rate of inflation, by these means reducing the real value of the debt and the tax needs of the government next period. This is the impulse-response version of the leit-motiv behind the covariance argument for nominal debt originally devised by Bohn (1988)⁶.

⁵I am assuming the VAR to be invertible.

⁶In the next section I provide a quantitative illustration of the welfare losses associated with this spike in government spending under different tax-smoothing assumptions.

Now suppose that the maturity of the debt is of n periods, $n > 1$. Then the objective of each minimization, regarding the $1/n$ fraction of the public debt which by assumption comes due each period, is not simply $\tau_t - E_{t-1}\tau_t$, but some function of the summation $\sum_{j=0}^{n-1} \tau_{t+j} - E_{t-1}\tau_{t+j}$. Put in terms of the impulse-response function, and still using a simplified version of the VAR, with g and π as the only variables, the new problem involves not only the contemporaneous response of inflation to a shock in government spending at time t , but also the subsequent terms $\pi_{t+j} - E_{t-1}\pi_{t+j}$, $j = 1, \dots, n-1$ as well.

In general, making $D(L) = B(L)C$ in (13):

$$x_{t+j} - E_{t-1}x_{t+j} = D_0\eta_{t+j} + D_1\eta_{t+j-1} + \dots + D_{j-1}\eta_{t+1} + D_j\eta_t$$

The important point to observe is that now $x_{t+j} - E_{t-1}x_{t+j}$ depends not only on the covariance matrix Σ , but also on the companion matrix of the coefficients of the VAR. To see this use the definition of D and write (13) as:

$$D^{-1}x_t = \eta_t$$

Suppose that in state-space notation this system is written as:

$$z_t = Az_{t-1} + H\eta_t, \quad E(\eta_t\eta_t') = I$$

In this case the impulse-response function⁷ $E_t x_{t+j} - E_{t-1}x_{t+j}$ reads C, AC, A^2C, \dots , for $j = 0, 1, 2, \dots$. Minimizing the non-expected value of taxes in a future period is now a problem of a completely different and much more complex nature, in which the covariance matrix Σ plays only a part of the game (not to mention the possibility of additional shocks to government spending in subsequent periods $j = 1, \dots, n-1$, a problem to which I have treated separately above, but which could be jointly analyzed here as well).

Is is difficult to imagine an administration of the public debt changing its whole composition each month (first problem), or even a $1/n$ fraction of it, based on such complicated calculations associated with n -period-ahead forecasts (second problem).

4 Overestimated Welfare Gains

⁷Note that if there are no other shocks after time t the impulse-response function can also be written as $x_{t+j} - E_{t-1}x_{t+j}$, for $j = 0, 1, 2, \dots$

In Goldfajn's model there is no possibility of intertemporal tax smoothing. For this reason, the optimal determination of the structure of the debt tends to have a higher impact over welfare gains, when compared to the case in which taxes are allowed to be optimally distributed through time. Since in the real world there is always the possibility of tax smoothing, the welfare gains of Goldfajn's one-period model are upward biased⁸.

In this section I draw upon Lucas and Stokey (1983) to illustrate this point quantitatively⁹. To simplify the calculations, the utility function is supposed to be quadratic and symmetric in (c) and leisure (x) . The representative consumer order preferences based on consumption (c) and leisure (x) by:

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, x_t) = \sum_{t=0}^{\infty} \beta^t (c_t + x_t - \frac{1}{2}c_t^2 - \frac{1}{2}x_t^2) \quad (14)$$

In each period the economy is endowed with one unit of time. Hence:

$$1 = c_t + x_t + g_t \quad (15)$$

where g_t denotes government spending at time t .

I want to investigate the case in which the economy faces one single shock in public spending in period zero. After the shock, spending resumes its previous level, equal to zero. Hence:

$$g_t = \bar{g} > 0, \quad t = 0$$

$$g_t = 0, \quad t > 0$$

The purpose of the exercise is to determine the respective percentage fall of consumption, taking as a reference the optimum (lump-sum) allocation, that makes the consumer have the same utility as that associated, respectively, with the Ramsey allocation (in which taxes are optimally chosen by the government, subject to the compatibility constraint $\sum_{t=0}^{\infty} \beta^t (U_c(t)c_t - U_x(t)(1 - x_t)) = 0$) and with the balanced-budget (*B.B.*) allocation. Since

⁸Note, though, that providing estimates of welfare gains is not among the objectives of Goldfajn's work.

⁹Note that Lucas and Stokey's economy is an economy with complete markets, in which the type of hedging argument considered by Bohn (1988) and his followers does not apply (see a discussion about that in the conclusions to this work). This point, though, does not affect the calculations here, since their purpose aims solely at illustrating the consequences of the absence of an optimal tax smoothing.

the algebra concerning this problem is well known and standard in the literature (see, e.g., Appendix A in Lucas and Stokey (1983), which deals with quadratic utilities as (14)) I omit the details here.

The results of the calculations for $\beta = .9$ and $\beta = .98$ are displayed in Table 1:

Table 1						(16)
Beta=.98			Beta=.9			
Consumption Loss (%)			Consumption Loss (%)			
\bar{g}	<i>B.B.</i>	<i>Ramsey</i>	\bar{g}	<i>B.B.</i>	<i>Ramsey</i>	
.2	0.32	0.23×10^{-2}	.2	1.59	0.06	
.15	0.98×10^{-1}	0.12×10^{-2}	.15	0.49	0.03	
.10	0.32×10^{-1}	0.04×10^{-2}	.10	0.16	0.01	

As one can notice from the table, welfare losses are much higher in the balanced-budget model than in the Ramsey model. Note also that the discrepancies are higher when $\beta = .9$ than in the case $\beta = .98$ (left side of the table). Also, a comparison between the *B.B.* loss with different betas shows that welfare losses are higher when β is lower. This happens because individuals with higher β have a relatively higher consumption at time zero, the point in time in which the balanced-budget hypothesis (in this case) generates the highest distortion.

Welfare gains derived from the covariance argument are overestimated under the assumption, as it happens in Goldfajn's model, that the budget is always balanced. Suppose for instance that, due to the covariance argument associated with the optimum denomination of the public debt, government expenditures in period zero happen to be (generously) reduced from 20% to just 10% of GDP. For $\beta = 0.98$, welfare gains accrued to hedging the debt are found to be 0.28% of GDP under Goldfajn's balanced-budget allocation but just 0.019% of GDP under Ramsey's allocation, around 14 times the Ramsey value. When $\beta = 0.9$ these values turn out to be 1.43% and 0.05%, over 28 times the Ramsey allocation.

The basic conclusion of this section is that possible welfare gains motivated by the covariance argument (which in Goldfajn's analysis implies a *pari-passu* reduction of the tax needs of the government) turn out to be much lower when optimum taxation is allowed.

5 Conclusions

In this work I have analyzed the model proposed by Goldfajn (2000) to study the choice of the composition of the public debt, focusing on some problems derived from its simplified time frame. Two points are worth mentioning as final remarks of this analysis: one of a theoretical nature, and another one of an applied nature, specifically related to the composition of the Brazilian debt.

Complete-market economies as those contemplated by Lucas and Stokey (1983) allow for a complete hedging under the realization of any state of nature. Shocks happen and taxes remain the same, there existing no need to change them. This is a well known characteristic of complete-market economies.

On the other hand, the main argument of Bohn (1988) and his followers, is that optimal structure of government debt must include some liabilities that are state contingent in real terms. The hedging argument, therefore, can be understood as an attempt to complete the markets and move towards the theoretical structure described by Lucas and Stokey. A usual view among administrators of the debt is that a more diversified asset structure allows for more risk sharing, moving towards the completion of the markets and, by these means, either keeping unchanged or, possibly, improving welfare.

It happens, though, that this type of argument has already been shown not to be valid in economies with two or more consumption goods. Though not directly related to government debt, a sequence of papers in the literature covers this issue.

Hart (1975) first constructed an example of a competitive economy in which the allocation with one single asset was Pareto dominated by the allocation obtained with no assets at all. Newberry and Stiglitz (1984) provided new examples in the context of international trade. Zame (1993) demonstrated that, unless one allows for default, having the number of assets tend towards infinity in economies with countably-infinite states does not guarantee Pareto optimality. Later work by Elul (1994) has shown that the type of phenomenon illustrated by Hart's example was indeed very general. In almost every incomplete-market economy with more than one consumption good and with sufficiently many uninsured states of nature, all agents could be made worse off by the introduction of an appropriate asset¹⁰. Cass and

¹⁰Elul (1994) also shows that in the same economies another asset can be found that

Citanna (1998) arrived at basically the same conclusions as Elul (1994)¹¹.

What all this line of work shows is that optimality characterized by (1) and (2) can be very myopic, in the sense that improving the government hedging may not be a desirable outcome from a Pareto-optimum perspective.

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makes all agents better off.

¹¹Curiously, although Elul's paper has been published four years before Cass and Citanna's, Elul (1994) cites Cass and Citanna, but Cass and Citanna (1998) do not cite Elul's result.

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